Dissertation Defense: "¹²⁹Xe Relaxation and Rabi Oscillations"

Mark Limes Oct. 30th, 2013



- Energy difference (Zeeman) is "resonated" by applying an oscillating field $\mu_S = \gamma S/\hbar$ $-\mu_S \cdot B \rightarrow E$ $E_{\perp} - E_{\uparrow} = \hbar \omega_0$



 For most MR exp., an oscillating field is perpendicular to a larger, quantizing field







In the high-field regime (B₀ >> B₁), use a rotating-wave approximation (RWA)



 $B_1' = 2B_1$

The magnetic fields and spin state can be visualized using a Bloch-sphere picture



Magnetic Resonance (MR) Considering a rotating-frame Bloch sphere (RFBS) elucidates spin dynamics $\hat{\mathbf{z}} |+1/2\rangle$ $\hat{\mathbf{z}} |+1/2\rangle$ ω_0 B_0 $\hat{\mathbf{x}}'$ $\omega = \omega_0$ $\omega = \omega_0$ $|-1/2\rangle$ $|-1/2\rangle$



– This nutation about B_{eff} is well-described quantitatively in a Rabi oscillation





NMR samples have many spins, thermally polarized, totaling a net magnetization M

 $P_0 = P_{\uparrow} - P_{\downarrow}$



 M is (conventionally) a pseudo-pure spin state, and only mimics a single spin state

Μ



 However, as a proper sum of its spin constituents, M obeys the same dynamics

Μ





 A typical, pulsed-NMR experiment relies on the function of a single coil



 The coil is first pulsed at the resonance frequency, causing a Rabi oscillation of M



Result of a 90°, or $\pi/2$, flip angle

 M is torqued about B₀, inducing an EMF, and generates a free induction decay (FID)



Decay results from dephasing of spins due to local environment (local field)







 A Fourier transform (FFT) of the FID gives a mapping to local field distributions





Hyperpolarization

 Spin-exchange optical pumping (SEOP) increases nuclear polarization by five orders of magnitude



Hyperpolarization

– This boost in signal allows for T_1 experiments to be run from the 'top down'





Average # of photons: 3/2

Average # of photons: 1



SEOP

 A highly polarized atomic spin-bath contacts with a nuclear spin-bath

$H = A\mathbf{I} \cdot \mathbf{S} + \gamma \mathbf{S} \cdot \mathbf{N} + \alpha \mathbf{K} \cdot \mathbf{S}$



T_1 : Solid ¹²⁹Xe Relaxation







 Trickle-freeze hyperpolarized xenon into sample chamber (Snow) held at 2 Tesla

- Set temperature of sample and perform T_1 experiment, with extremely small flip angles (approx. 1-8°)

- Able to take snow through liquid phase, and refreeze at 77 K (Ice)



- See a substantial difference between ice and snow relaxation times at 77 K



Time (min.)

– Difference is reproducible across many

cot_11nc		T_1 (min.)	Probe	Flip Angle	Buffer Gas
sel-ups		169.93 ± 0.25	Tank	N/A	Yes
-		168.16 ± 0.16	Tank	N/A	No
		165.14 ± 0.70	Tank	N/A	Yes
		168.7 ± 1.0	Tank	N/A	Yes
	Ice	173.6 ± 1.5	Tank	N/A	Yes
		171.44 ± 0.32	Flat	$2.67^\circ\pm0.19^\circ$	Yes
		172.02 ± 0.31	Flat	$3.11^\circ\pm0.30^\circ$	No
		166.55 ± 0.81	Flat	$3.02^\circ\pm0.07^\circ$	No
		167.30 ± 0.33	Flat	$3.31^\circ\pm0.32^\circ$	Yes
	Ice Average	169.2 ± 1.2			

77 K

	150.79 ± 0.43	Tank	N/A	No
	148.47 ± 0.28	Tank	N/A	Yes
	150.80 ± 0.25	Tank	N/A	Yes
	148.61 ± 0.42	Flat	$8.96^\circ\pm0.05^\circ$	No
Snow	149.30 ± 0.40	Flat	$6.27^\circ\pm0.07^\circ$	Yes
	149.80 ± 0.40	Flat	$6.91^\circ \pm 0.14^\circ$	Yes
	150.19 ± 0.37	Flat	$6.91^\circ\pm0.14^\circ$	Yes
	151.02 ± 0.51	Flat	$4.67^\circ\pm0.22^\circ$	No
Snow Average	149.87 ± 0.54			



– Intrinsic difference in ¹²⁹Xe T_1 between bulk snow and ice

- Theory predicts no difference between polycrystalline and single crystal $^{129}\rm{Xe}~T_1$
- Initial results at 77 K indicate ice T_1 is *longer* than theory suggests




– Spin-rotation is dominate gas mechanism

 129 Xe-Xe

Spin-rotation is dominate gas mechanism

THEORY OF QUADRUPOLAR NUCLEAR SPIN-LATTICE RELAXATION

by J. VAN KRANENDONK

PHYSICAL REVIEW B

VOLUME 59, NUMBER 13

1 APRIL 1999-I

¹²⁹Xe spin relaxation in frozen xenon

R. J. Fitzgerald,* M. Gatzke,[†] David C. Fox,[‡] G. D. Cates, and W. Happer Department of Physics, Princeton University, Princeton, New Jersey 08544 (Received 2 September 1998)

(Received 2 September 1998)

VOLUME 18, NUMBER 17

PHYSICAL REVIEW LETTERS

24 April 1967

THEORY OF QUADRUPOLAR NUCLEAR SPIN-LATTICE RELAXATION DUE TO ANHARMONIC RAMAN PHONON PROCESSES

J. Van Kranendonk and M. Walker* Department of Physics, University of Toronto, Toronto, Canada (Received 12 January 1967, revised manuscript received 17 February 1967)

Direct Process (DP) Linear temp. dependence, Field dependent Harmonic Raman (1R) Quad. temp. dependence, Field independent **d** Anharmonic Raman (aR) Quad. temp. dependence, Field independent

-Van Kranendonk, Walker, PRL, (1967)

Direct Process (DP) Linear temp. dependence, Field dependent Harmonic Raman (1R) Quad. temp. dependence, Field independent



Anharmonic Raman (aR) Quad. temp. dependence, Field independent

Too weak/improbable

-Van Kranendonk, Walker, PRL, (1967)

Theory: Solid ¹²⁹Xe Relaxation – Direct processes less probable than two-

phonon processes XENON 2.0 IOK Debye approx. (v) (ARBITRARY UNITS) Energy of Phonon 2 1.6 spin flip 1.2 Phonon 1 0.8 0.4

FIG. 2. Density-of-states function for xenon at 10 K, normalized to unit area.

0.6

0.0

0.2

0.4

–Lurie, et al., PRB (1974)

1.2

1.4

1.0

ο.8 ν (THz)

– 1R, Fitzgerarld, et al., PRB (1999)

$$\frac{c_{K0}}{h} = \left(\frac{\mu_K}{K\mu_B}\right) \left(\frac{\hbar}{8\pi M R_0^2}\right) (\sigma_g - \sigma_c)$$

$$\frac{c_{K0}}{h} = -27 \text{ Hz}$$

$$\nu = \frac{c_K}{\hbar} \mathbf{K} \cdot \mathbf{I} \cdot \boldsymbol{\omega} = c_K \mathbf{K} \cdot \mathbf{N}$$

$$\mathbf{K} = \frac{\delta_1}{\mathbf{K}}$$

Theory: Solid ¹²⁹Xe Relaxation
- 1R, Fitzgerarld, et al., PRB (1999)

$$\frac{1}{T_1^S} = \underbrace{\frac{9\hbar^2 \mu_K^2 (\sigma_g - \sigma_c)^2}{256\pi K^2 \mu_B^2 M^2 R_0^2 T_D^2}}_{\sum_m g_m} \sum_m g_m \left(4 + \frac{8}{3}\epsilon_0 + \frac{1}{3}\epsilon_0^2 (2\cos^2\theta_m + 1)\right)$$

$$\times \int_0^1 du \ u^4 \underbrace{\frac{e^{uT_D}}{(e^{uT_D} - 1)^2}}_{\text{Temperature dependence}} \underbrace{\left[1 + \operatorname{sinc} \left(u\sigma_m (6\pi^2 \sqrt{2})^{\frac{1}{3}}\right) + \operatorname{sinc} \left(u(6\pi^2 \sqrt{2})^{\frac{1}{3}}\right)\right]^2}_{\text{Conservation of momentum}}$$

$$\frac{c_{K0}}{h} = -27 \text{ Hz}$$

Spin-rotation coupling strength

Hanni, $c_{K0}/h = -16$ Hz

- Fermi's golden rule $dW_{fi} = \frac{2\pi}{\hbar} |\nu_{fi}|^2 \rho(E_e) d\Omega_e$

$$\nu = \sum_{\beta} c_K(R_{\beta\alpha}) \mathbf{N}_{\beta\alpha} \cdot \mathbf{K}$$
$$= \frac{1}{2\hbar} \sum_{\beta} c_K(R_{\beta\alpha}) \mathbf{R}_{\beta\alpha} \times \mathbf{P}_{\beta\alpha} \cdot \mathbf{K}$$

- Fermi's golden rule $dW_{fi} = \frac{2\pi}{\hbar} |\nu_{fi}|^2 \rho(E_e) d\Omega_e$ $= \frac{1}{2\hbar} \sum_{\beta} c_K(R_{\beta\alpha}) \mathbf{R}_{\beta\alpha} \times \mathbf{P}_{\beta\alpha} \cdot \mathbf{K}$ $= \frac{1}{2\hbar} \sum_{\beta} c_K(R_{\beta\alpha}) \mathbf{R}_{\beta\alpha} \times \mathbf{P}_{\beta\alpha} \cdot \mathbf{K}$ $|i\rangle = |m_K = 1/2; \dots, n_{\mathbf{k}_a j_a}, n_{\mathbf{k}_e j_e}, \dots\rangle,$ $|f\rangle = |m_K = -1/2; \dots, n_{\mathbf{k}_a j_a} - 1, n_{\mathbf{k}_e j_e} + 1, \dots\rangle$



 At high temp., temp. dependence comes from only phonon occupation numbers, n

$$|\langle n_i - 1, n_j + 1 | a_i a_j^{\dagger} | n_i, n_j \rangle|^2 = n_i (n_j + 1)$$

Fermi's golden rule

 These occupation numbers are averaged over using a Bose-Einstein distribution

$$< n_i > < n_j + 1 > = \frac{1}{e^{x/T} - 1} \frac{e^{x/T}}{e^{x/T} - 1}$$

– Which appears exactly in T_1 formula

$$\frac{1}{T_1^S} = \frac{9\hbar^2 \mu_K^2 (\sigma_g - \sigma_c)^2}{256\pi K^2 \mu_B^2 M^2 R_0^2 T_D^2} \sum_m g_m \left(4 + \frac{8}{3}\epsilon_0 + \frac{1}{3}\epsilon_0^2 (2\cos^2\theta_m + 1) \right) \\ \times \int_0^1 du \ u^4 \frac{e^{\frac{uT_D}{T}}}{\left(e^{\frac{uT_D}{T}} - 1\right)^2} \left[1 + \operatorname{sinc}\left(u\sigma_m (6\pi^2\sqrt{2})^{\frac{1}{3}} \right) + \operatorname{sinc}\left(u(6\pi^2\sqrt{2})^{\frac{1}{3}} \right) \right]^2$$

$$< n_i > < n_j + 1 > = \frac{1}{e^{x/T} - 1} \frac{e^{x/T}}{e^{x/T} - 1}$$

At high temp. (higher than Debye temp., 55 K), each unique *n* contributes *T*

$$< n_i > < n_j + 1 > = \frac{1}{e^{x/T} - 1} \frac{e^{x/T}}{e^{x/T} - 1}$$

 $\rightarrow \frac{1}{1 + x/T - 1} \frac{1}{1 + x/T - 1} \propto T * T$

 The third phonon's occupation number in the aR process drops out, T² remains



Irrespective of mechanism, any *generic* process has a temp. dependence defined
 by the number of unique external phonons



 Though energy conservation is violated, consider non-unique phonon case



 $|\langle n| a^{\dagger}a |n\rangle|^2 = n^2$

Fermi's golden rule

$$\langle n^2 \rangle = \langle n \rangle + 2 \left< n \right>^2 \propto T + 2T^2$$
 (High-temp.)

 In scattering problems, conservation of momentum is more strict than energy

$$W_{fi} = \frac{2\pi}{\hbar} \sum_{j_e j_a} \int_0^{E_D} dE_a d\Omega_a d\Omega_e ||v_{fi}|^2 \rho^2(E_a)$$

 Typically in phonon scattering, a sum over entire lattice generates a delta function

$$\frac{1}{N} \sum_{l} e^{i\mathbf{r}^{l} \cdot (\mathbf{k_{1}} - \mathbf{k_{2}})} = \Delta(\mathbf{k_{1}} - \mathbf{k_{2}})$$
$$\Delta(\mathbf{k}) = \begin{cases} 1 & \text{if } \mathbf{k} = \mathbf{G}, \\ 0 & \text{otherwise.} \end{cases}$$

Momentum conservation leads to a sizable difference

 $\cdots \Delta(\mathbf{k})$

 $T_1 = 300 \text{ min.}$ $T_1 = 1500 \text{ min.}$

(Using Hanni/*h*, $c_{K0} = -16$ Hz)

Open: Solid ¹²⁹Xe Relaxation

- Much future work, reopens field
- Create a structure-dependent theory
- Determine conservation of momentum
- Get lower temp. ice data, single crystal(?)
- Snow, experiments and theory

Finished: Solid ¹²⁹Xe Relaxation

- Developed ice method that allows for unprecedented, robust T_1 data
- Discovered intrinsic difference between bulk snow and ice
- Found generic high-temp. T_1 behavior
- Found structural dependence in dilute T_2

Unconventional MR

 Optically Detected MR, Electrically Detected MR; dipolar and exchange

 Permutation symmetry of paramagnetic pairs determines signal, not polarization

$$\begin{array}{c} |\uparrow\uparrow\rangle\\ \cos(\phi) \mid\uparrow\downarrow\rangle - \sin(\phi) \mid\downarrow\uparrow\rangle\\ \cos(\phi) \mid\uparrow\downarrow\rangle + \sin(\phi) \mid\downarrow\uparrow\rangle\\ \mid\downarrow\downarrow\rangle\end{array}$$

 Resonate spin-1/2 pair, detect the optical or electrical deviation from steady state

 ${\it Spin \ manipulation_pulse}$





Repeat for detuned pulses



 Implemented Liouville-space formalism, allowing for quick computation of inhomogeneous, stochastic Liouville equation, for dipolar and exchange coupled pairs

$$\rho = \frac{|\uparrow\rangle}{|\downarrow\rangle} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \qquad \rho = \begin{pmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{21} \\ \rho_{21} \end{pmatrix}$$

Generation takes 10 min. instead of 7 days.



Limes, et al., PRB (2013)

Showed that single-transition Rabi frequencies are given by

 $\Omega_{ij} = \sqrt{(1 \mp \sin 2\phi) (\gamma B_1)^2 + (\omega - \omega_{ij})^2}$ and with sufficient dipolar coupling, $\sin 2\phi \rightarrow -1$

leading to an on-resonant Rabi frequency $\sqrt{2}\gamma B_1$

Mixed MR



- Readjust for comfort, quantization x'-axis



- Access unconventional regimes, $B_2 >> B_1$



 B_2'

Relatively high polarization

– Determine Rabi envelope dynamics $B_1 >> B_2$



Regime: weak-resonant modulation





Acknowledgements/Work


Acknowledgements/Home





Acknowledgements/Heidi





